Online Appendix
“Pandering and Electoral Competition”

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1 Derivation of Equation (A.5)

Begin with

$$\mu(\theta) \quad = \quad \frac{\sigma_n(e \mid \theta)\,\sigma_n(e' \mid \theta)\Pr(\theta)}{\sum_{\hat{\theta}\in\Theta}\sigma_n\left(e \mid \hat{\theta}\right)\sigma_n\left(e' \mid \hat{\theta}\right)\Pr(\theta)}.$$

First, let $\theta \notin \Theta_e \cup \Theta_{e'}$. Then we have

$$\mu(\theta) = \lim_{n\to\infty} \frac{\kappa_{e^*(\theta),e'}\kappa_{e^*(\theta),e}\epsilon^{2n}\Pr(\theta)}{\left[\begin{array}{c} \sum_{\hat{\theta}\notin\Theta_e\cup\Theta_{e'}} \kappa_{e^*\left(\hat{\theta}\right),e}\kappa_{e^*\left(\hat{\theta}\right),e'}\epsilon^{2n}\Pr\left(\hat{\theta}\right) + \\ \sum_{\hat{\theta}\in\Theta_e}\left(1-\epsilon^n\sum_{z\neq e}\kappa_{e,z}\right)\kappa_{e,e'}\epsilon^n\Pr\left(\hat{\theta}\right) + \\ \sum_{\hat{\theta}\in\Theta_{e'}}\left(1-\epsilon^n\sum_{z\neq e'}\kappa_{e',z}\right)\kappa_{e',e}\epsilon^n\Pr\left(\hat{\theta}\right) \end{array}\right]}$$

Dividing both numerator and denominator by ϵ^n (note that ϵ^n is fixed and can be taken to common factor of the sums) and taking the limit as $n \to \infty$, we get $\mu(\theta) = 0$.

Then, let $\theta \in \Theta_e$. We have

$$\mu(\theta) = \lim_{n\to\infty} \frac{\left(1-\epsilon^n \sum_{e'\neq e} \kappa_{e,e'}\right) \kappa_{e,e'} \epsilon^n \Pr(\theta)}{\left[\begin{array}{r} \sum_{\hat{\theta}\notin\Theta_e\cup\Theta_{e'}} \epsilon^{2n} \kappa_{e^*(\hat{\theta}),e} \kappa_{e^*(\hat{\theta}),e'} \Pr\left(\hat{\theta}\right) + \\ \sum_{\hat{\theta}\in\Theta_e} \left(1-\epsilon^n \sum_{z\neq e} \kappa_{e,z}\right) \kappa_{e,e'} \epsilon^n \Pr\left(\hat{\theta}\right) + \\ \sum_{\hat{\theta}\in\Theta_{e'}} \left(1-\epsilon^n \sum_{z\neq e'} \kappa_{e',z}\right) \kappa_{e',e} \epsilon^n \Pr\left(\hat{\theta}\right) \end{array}\right]}$$

Divide both numerator and denominator by the numerator and simplify (note that ϵ^n is fixed and can be taken to common factor of the sums):

$$\mu(\theta) \quad = \quad \lim_{n\to\infty} \left[Q_1 + Q_2 + Q_3\right]^{-1}$$

where

$$Q_1 \quad \equiv \quad \frac{\epsilon^n \sum_{\hat{\theta}\notin\Theta_e\cup\Theta_{e'}} \kappa_{e^*(\hat{\theta}),e} \kappa_{e^*(\hat{\theta}),e'} \Pr\left(\hat{\theta}\right)}{\left(1-\epsilon^n \sum_{z\neq e} \kappa_{e,z}\right) \kappa_{e,e'} \Pr(\theta)};$$

$$Q_2 \equiv \frac{\sum_{\hat{\theta}\in\Theta_e} \left(1-\epsilon^n \sum_{z\neq e} \kappa_{e,z}\right) \kappa_{e,e'} \Pr\left(\hat{\theta}\right)}{\left(1-\epsilon^n \sum_{z\neq e} \kappa_{e,z}\right) \kappa_{e,e'} \Pr(\theta)};$$

$$Q_3 \equiv \frac{\sum_{\hat{\theta}\in\Theta_{e'}} \left(1-\epsilon^n \sum_{z\neq e'} \kappa_{e',z}\right) \kappa_{e',e} \Pr\left(\hat{\theta}\right)}{\left(1-\epsilon^n \sum_{z\neq e} \kappa_{e,z}\right) \kappa_{e,e'} \Pr(\theta)}.$$

Taking the limit as $n \to \infty$, $Q_1 \to 0$ and

$$\begin{aligned}
\mu(\theta) &= \left[\frac{\kappa_{e,e'} \sum_{\hat{\theta} \in \Theta_e} \Pr\left(\hat{\theta}\right)}{\kappa_{e,e'} \Pr(\theta)} + \frac{\kappa_{e',e} \sum_{\hat{\theta} \in \Theta_{e'}} \Pr\left(\hat{\theta}\right)}{\kappa_{e,e'} \Pr(\theta)} \right]^{-1} \\
&= \left[\frac{\Pr(\Theta_e)}{\Pr(\theta)} + \frac{\kappa_{e',e} \Pr(\Theta_{e'})}{\kappa_{e,e'} \Pr(\theta)} \right]^{-1}.
\end{aligned}$$

That is,

$$\mu(\theta) = \frac{\Pr(\theta)}{\Pr(\Theta_e) + \frac{\kappa_{e',e}}{\kappa_{e,e'}} \Pr(\Theta_{e'})} \text{ for all } \theta \in \Theta_e.$$

Very similar steps lead to

$$\mu(\theta') = \frac{\Pr(\theta')}{\Pr(\Theta_{e'}) + \frac{\kappa_{e,e'}}{\kappa_{e',e}} \Pr(\Theta_e)} \text{ for all } \theta' \in \Theta_{e'}.$$

2 Equilibrium Perfection

Sequential Equilibrium puts restrictions on the beliefs of the voters such that these beliefs are robust to vanishingly small trembles of the candidates. The Proof of Proposition 1 in the paper uses of a sequence of independent trembles of the candidates. Thus, as shown by Kohlberg and Reny (1997), the equilibrium beliefs are robust to independent trembles of the cadidates. This means that there is a sequence of trembles inducing beliefs (by Bayes' rule) that *in the limit* approach the equilibrium beliefs. What this solution concept does not guarantee is that *along the sequence*, the beliefs are sufficient to induce the same best response for the voters, therefore inducing the same behavior in the candidates. The solution concept which appropriately takes care of this is Perfect Equilibrium. The fully revealing equilibria in Section 3 of the paper are perfect. Also, pooling equilibria on any policy that is not the worst for the voters in at least one state are perfect. In this section, we show these results for a simple case with two states, θ and θ', and two policies, e and e'. Very similar arguments extend to the general model of Section 2 of the paper.

2.1 Perfection of Equilibria in Section 3

In the fully revealing equilibria of Section 3, whenever the two proposals are different, voters' beliefs are interior: voters give positive probability to all states in which one of the proposals is played. As shown in Section 3, generally in equilibrium there exists a signal s^* such that voters who observe s^* are indifferent between the two proposals. When this is the case, by Lemma 1 and Part 1 of the proof of Proposition 1, candidates are induced to propose the optimal policy for the voters in each state.

In the Proof of Proposition 1, we show that there exists a sequence of completely mixed candidates' strategies with the following properties: (i) the ratio between the probability that a candidate deviates when policy e is optimal and the probability that she deviates when $e' \neq e$ is optimal has a positive and finite limit; and (ii) the limit is exactly such that voters who observe s^* are indifferent between the two policies. To show that this is a perfect equilibrium we need to show that property (ii) holds not only at the limit of the sequence, but also for all n greater than some finite N. That is, the ratio between the two deviation probabilities must be constant when approaching its limit. Indeed, if this is the case, we know that the equilibrium strategies of the voters are also a best response to small trembles of the candidates.

Let voters with signal s^* be indifferent between proposals e and e' if[1]

$$\mu^*(\theta) = \left(1 + \kappa \frac{\Pr(\theta')}{\Pr(\theta)}\right)^{-1}.$$

We want to show that there is a sequence of completely mixed strategies indexed by n such that the beliefs induced along the sequence, $\mu(\theta)_n$ are equal to $\mu^*(\theta)$ for all n greater than some N.

Let $\sigma_n(e' \mid \theta) = \epsilon(n)$ and $\sigma_n(e \mid \theta') = \kappa\epsilon'(n)$, $\epsilon, \epsilon' : \mathbb{N} \to \mathbb{R}_{++}$, $\lim_{n\to\infty} \epsilon(n) = \lim_{n\to\infty} \epsilon'(n) = 0$. Then,

$$\mu(\theta) = \lim_{n\to\infty} \frac{(1-\epsilon(n))\,\epsilon(n)\Pr(\theta)}{(1-\epsilon(n))\,\epsilon(n)\Pr(\theta) + (1-\kappa\epsilon'(n))\,\kappa\epsilon'(n)\Pr(\theta')}.$$

Dividing by the numerator:

$$\mu(\theta) = \lim_{n\to\infty}\left[1 + \frac{(1-\kappa\epsilon'(n))\,\epsilon'(n)}{(1-\epsilon(n))}\frac{\kappa\Pr(\theta')}{\Pr(\theta)}\right]^{-1}.$$

[1]This formulation is consistent with Lemma 1.

Clearly $\mu(\theta) = \mu^*(\theta)$ if

$$\lim_{n \to \infty} \frac{(1 - \kappa \epsilon'(n)) \epsilon'(n)}{(1 - \epsilon(n))} = 1.$$

Hence, for the equilibrium to be perfect we need that this fraction is equal to 1 also along the sequence:

$$\epsilon(n) = 1 - (1 - \kappa \epsilon'(n)) \epsilon'(n)$$

which only imposes some restrictions on ϵ and ϵ'.

In conclusions, as expected, not all sequences of completely mixed strategies that give the correct beliefs are constant along the sequence, but there is enough freedom to pick an appropriate one that does. The same can be replicated, though with much more algebra, for the case of many states and policies. One should notice that one of the proposals can be a policy that is never optimal. When this is the case, the sequence converges to equilibrium beliefs such that voters have strict preferences. This means that there exists N such that, for $n > N$, voters have the same strict preferences along the sequence.

2.2 Other Equilibria

We mentioned in the paper that there exist a plethora of pooling equilibria. For example, when there are two policies and two states, both candidates proposing e and voters always voting for the candidate proposing e whenever the candidates make different proposals is a sequential equilibrium. In equilibrium, voters believe that a tremble is infinitely more likely in state θ' than in state θ. Hence, whenever a candidate deviates, they vote against her.

This equilibrium is also perfect. Indeed, voters' beliefs induce strict preferences between the policies. It follows that there exists N such that, for $n > N$, voters have the same strict preferences along the sequence. Indeed, Kreps and Wilson (1982) show that all strict sequential equilibria are perfect and our pooling equilibrium is a strict sequential equilibrium.

References

Kohlberg, E. and Reny, P. J. (1997) Independence on relative probability spaces and consistent assessments in game trees, *Journal of Economic Theory*, **75**, 280–313.

Kreps, D. M. and Wilson, R. (1982) Sequential equilibria, *Econometrica*, **50**, 863–894.